LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc.** DEGREE EXAMINATION – **STATISTICS**

FIFTH SEMESTER – NOVEMBER 2012

# ST 5504 - ESTIMATION THEORY

 Date : 01/11/2012 Dept. No. Max. : 100 Marks

 Time : 9:00 - 12:00

**PART - A**

**Answer ALL the questions: (10 x 2 = 20)**

1. Define Unbiasedness.
2. If T is an unbiased estimator of θ, show that T2 is a biased estimator for θ2.
3. Define Efficiency.
4. Let X1, X2, …, Xn be a random sample from a population with pdf

f( x, θ) = θ x θ – 1, 0 < x < 1, θ > 0.

Show that  is sufficient for θ.

1. Define BLUE.
2. What is meant by prior and posterior distribution?
3. Define sufficiency.
4. Write down the normal equation associated with a simple regression model.
5. Define Completeness.
6. Define MVB.

**PART- B**

**Answer any FIVE questions: (5 x 8 = 40)**

1. State and prove the sufficient condition for an estimator to be consistent.
2. Let X1, X2, …, Xn be a random sample from a Bernoulli distribution:

 $f\left(x, θ\right)= \left\{\begin{matrix}θ^{x}(1- θ)^{1-x} ;x=0, 1 \\ 0 otherwise\end{matrix}\right\}$

Show that  is a complete sufficient statistics for θ.

1. Mention the properties of MLE.
2. A random sample X1, X2, X3, X4, X5 of size 5 is drawn from a normal population with unknown mean µ. Consider the following estimators to estimate µ:

(i)  (ii) $t\_{2}= \left(\frac{X\_{1}+X\_{2}}{2}\right)+ X\_{3}$ (iii)  where λ is

 such that t3 is an unbiased estimator of μ.

1. Find λ.
2. Are t1 and t2 unbiased?
3. Which of the three is the best estimator?
4. State and prove Cramer – Rao Inequality.
5. Samples of sizes n1 and n2 are drawn from two populations with mean T1 and T2 and common variance σ2. Find the BLUE of l1T1 + l2T2.
6. Prove that if T1 and T2 are UMVUE of g(θ) then T1 = T2 almost surely.
7. Obtain the UMVUE of the parameter λ for the poisson distribution based on a random sample of size n.

**PART - C**

**Answer any TWO questions: (2 x 20 = 40)**

1. a) State and prove Rao – Blackwell theorem.

b) Show that if the MLE exists uniquely then it is a function of the sufficient statistic.

1. a) State and prove the necessary and sufficient for a parametric function to be linearly

 estimable.

b) Prove that the MLE of α of a population having density function: 0 < x < α

 for a sample of size one is 2x, x being the sample value. Show also that the estimate is

 biased.

1. a) State and prove factorization theorem.

b) Let (X1, X2, …, Xn) be a random sample from N(µ, σ2) . Obtain the Cramer – Rao

 lower bound for the unbiased estimator of μ.

1. a) Explain the method of moments.

b) Let X1, X2, …, Xn be a random sample from Bernoulli distribution b(1, θ). Obtain the

 Bayes estimator for θ by taking a suitable prior.

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